Multiphase Flow and Heat Transfer

ME546

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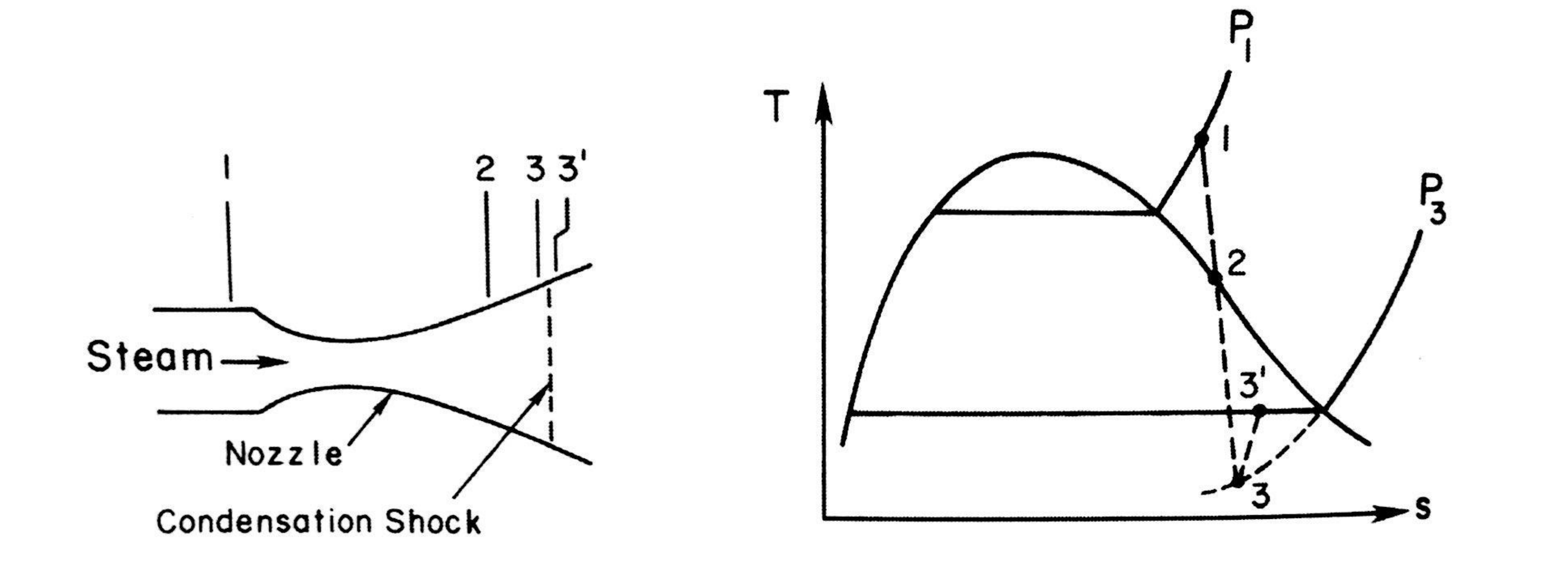
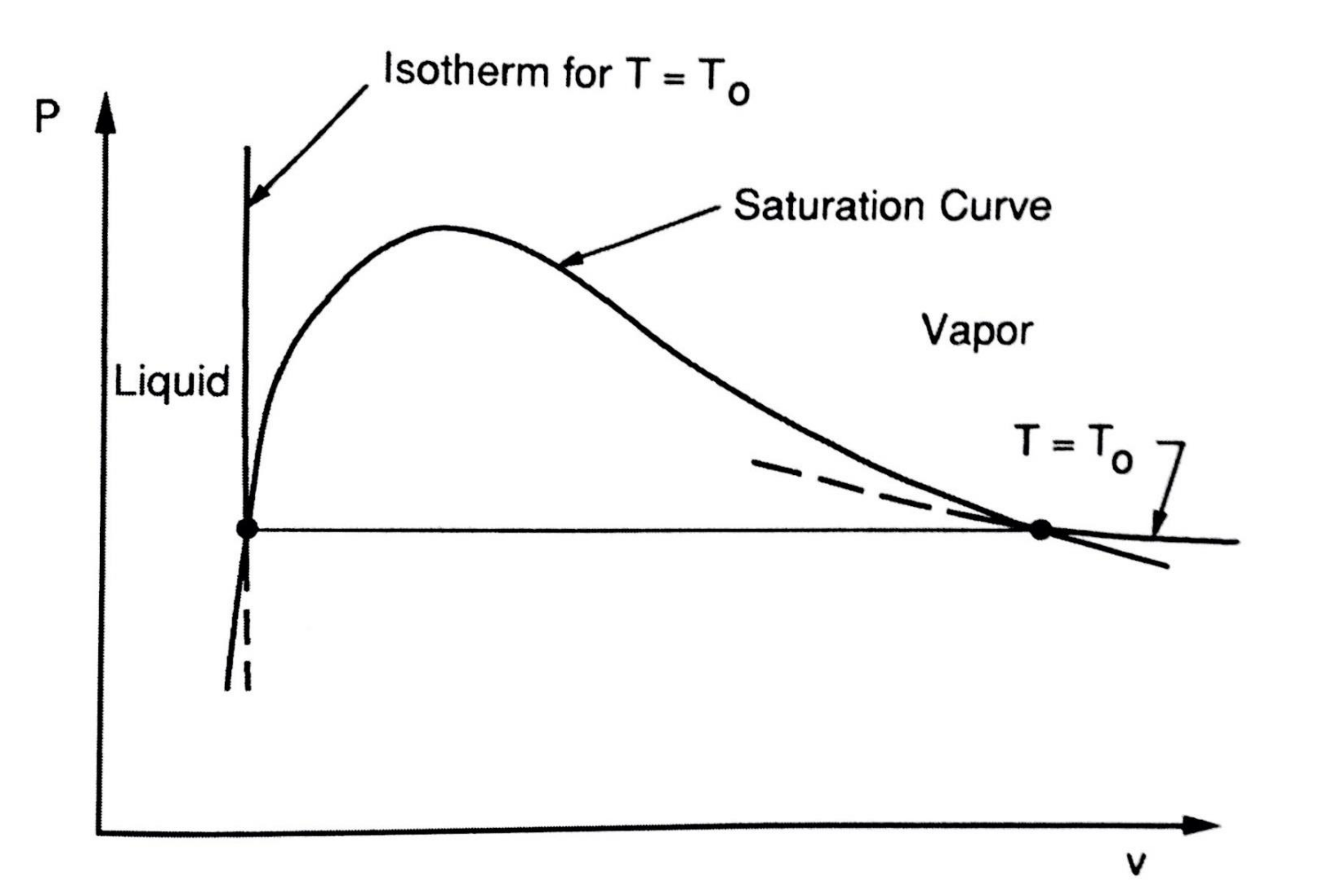
Metastable State

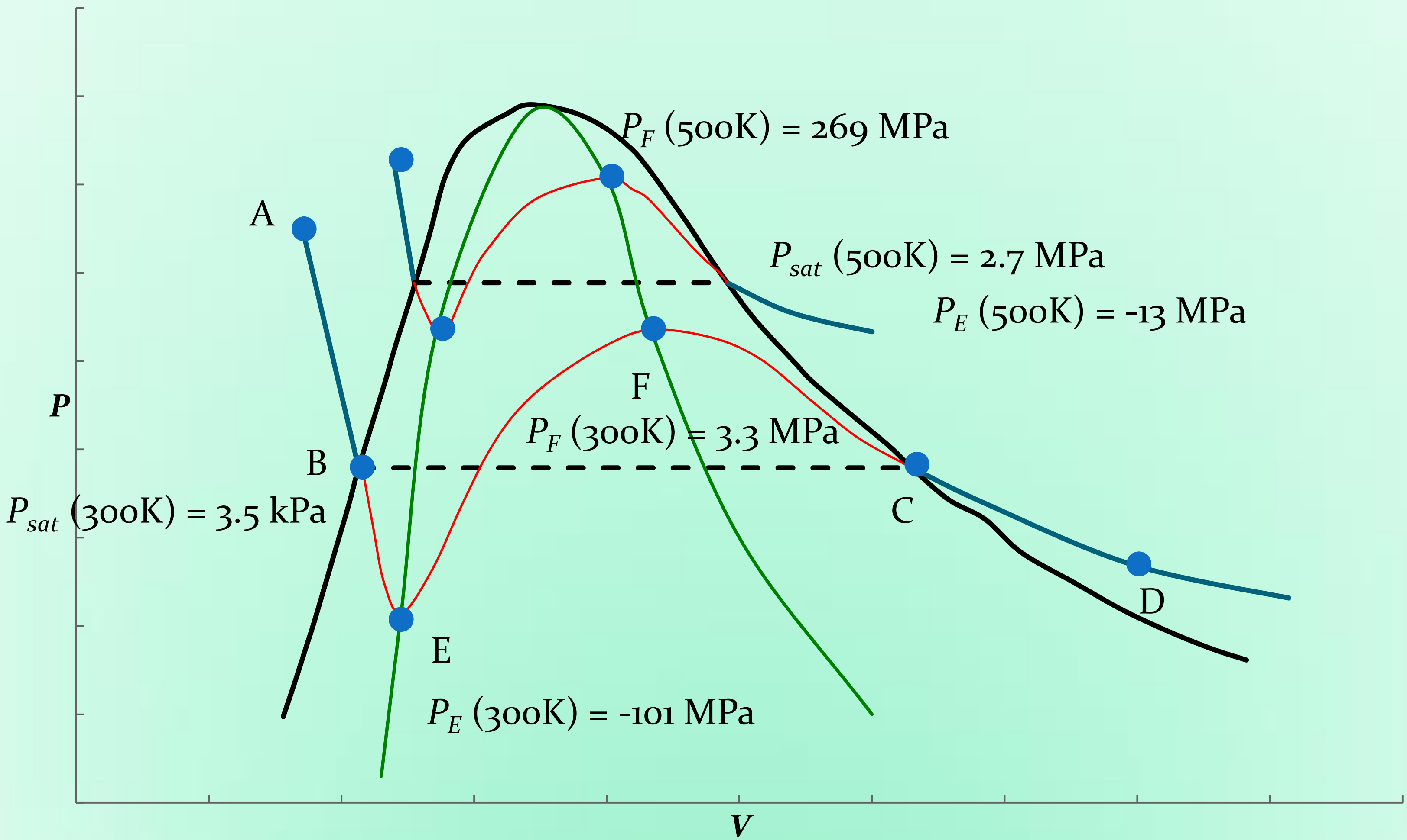
Vapor that is supercooled below its equilibrium saturation temperature and liquid that is superheated above its equilibrium saturation temperature exist in a non-equilibrium condition referred to as a **metastable state**.



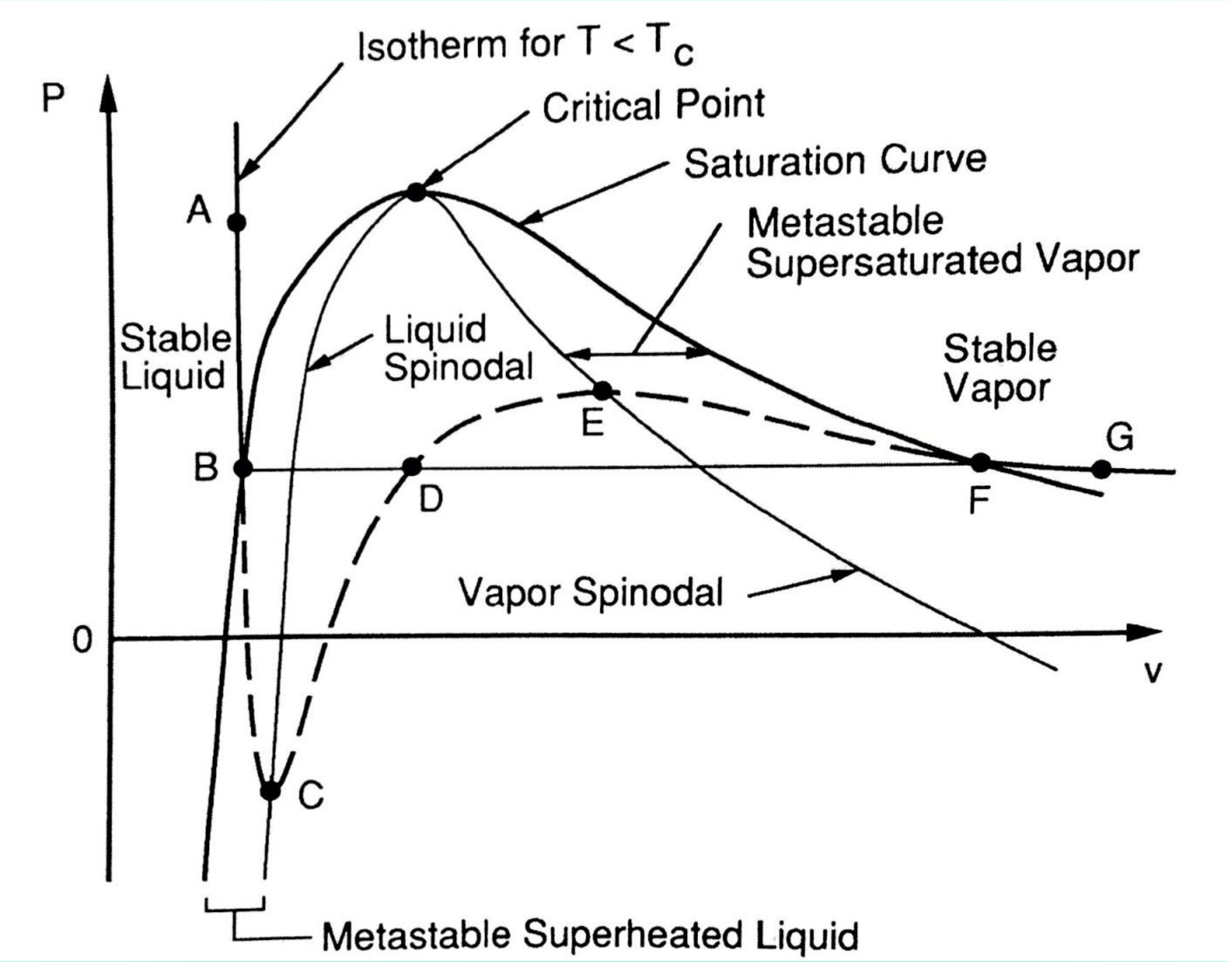
Metastable State



Compressing water (*l*) isothermally



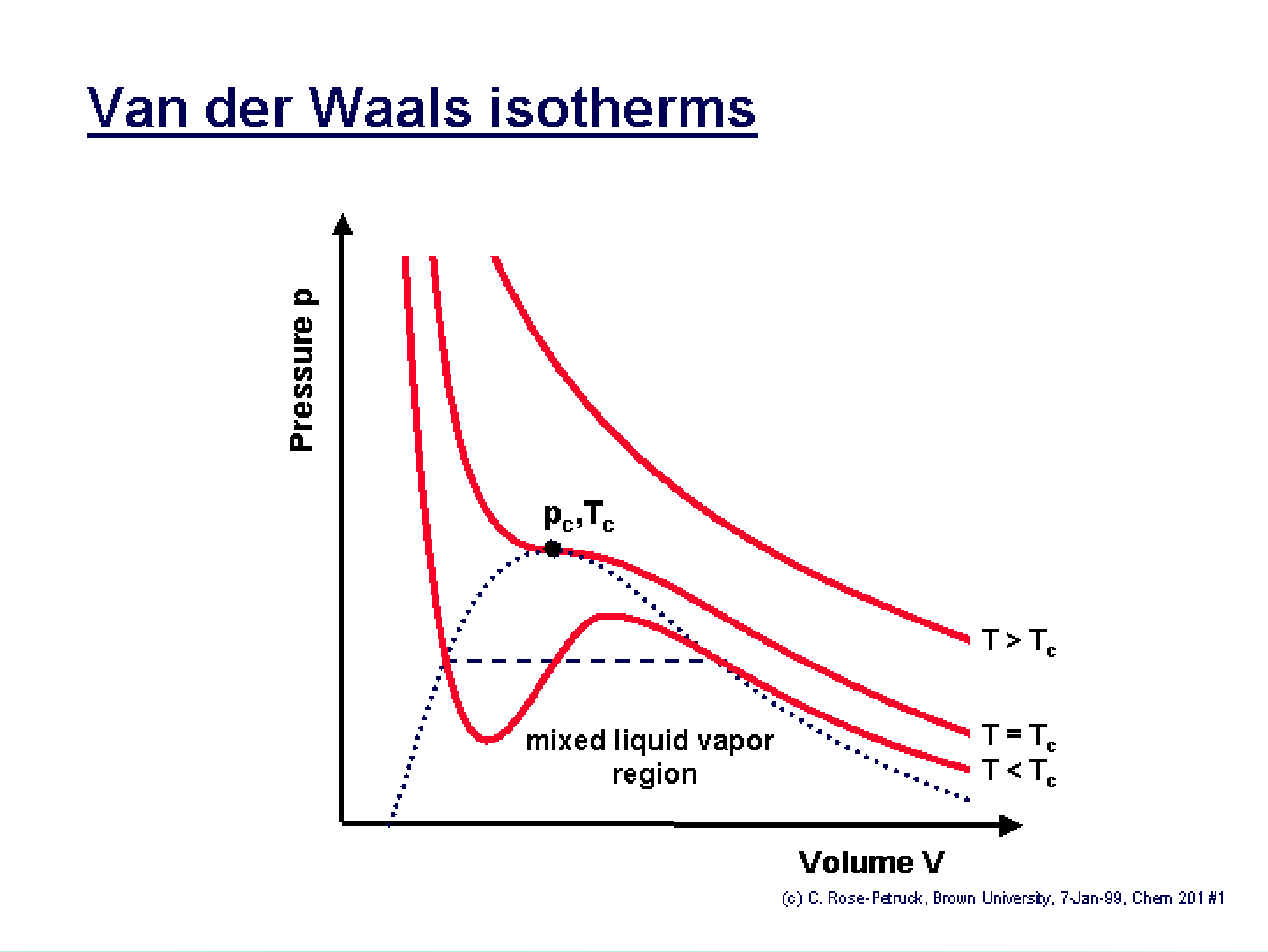
Metastable Regions – Spinodal Points



Van der Waals Equation

$$P = \frac{RT}{v - b} - \frac{a}{v^2}$$

$$P_r = \frac{8T_r}{3v_r - 1} - \frac{3}{v_r^2}$$



Reduced properties, $P_r = \frac{P}{P_c}, v_r = \frac{v}{v_c}, T_r = \frac{T}{T_c}$

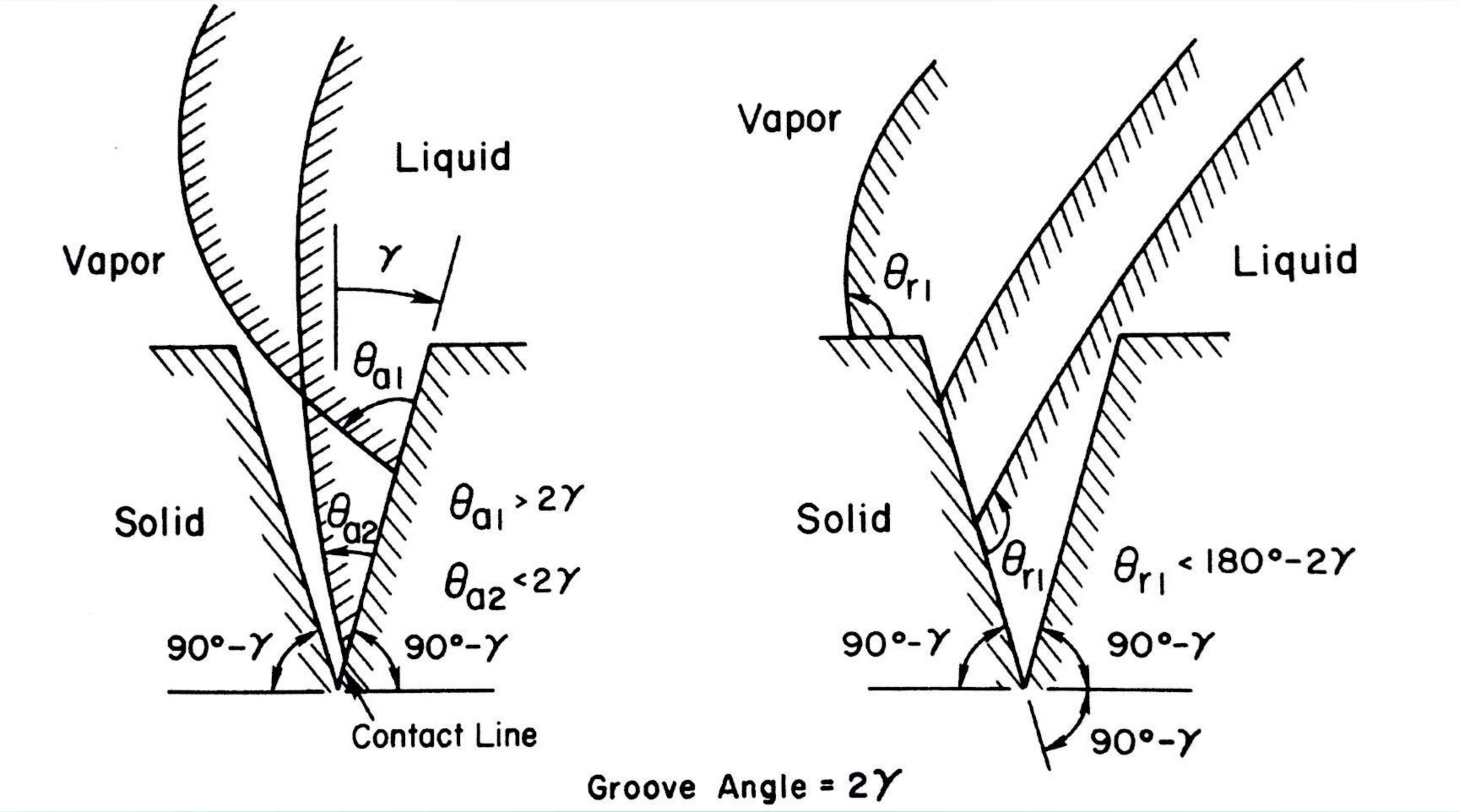
Derive the relation that specifies the spinodal conditions as predicted by the van der Waals equation. Use this result to determine the theoretical limit of superheat for nitrogen at atmospheric pressure. T_c = 126.3 K, P_c = 3396 kPa

Tr = 0.848

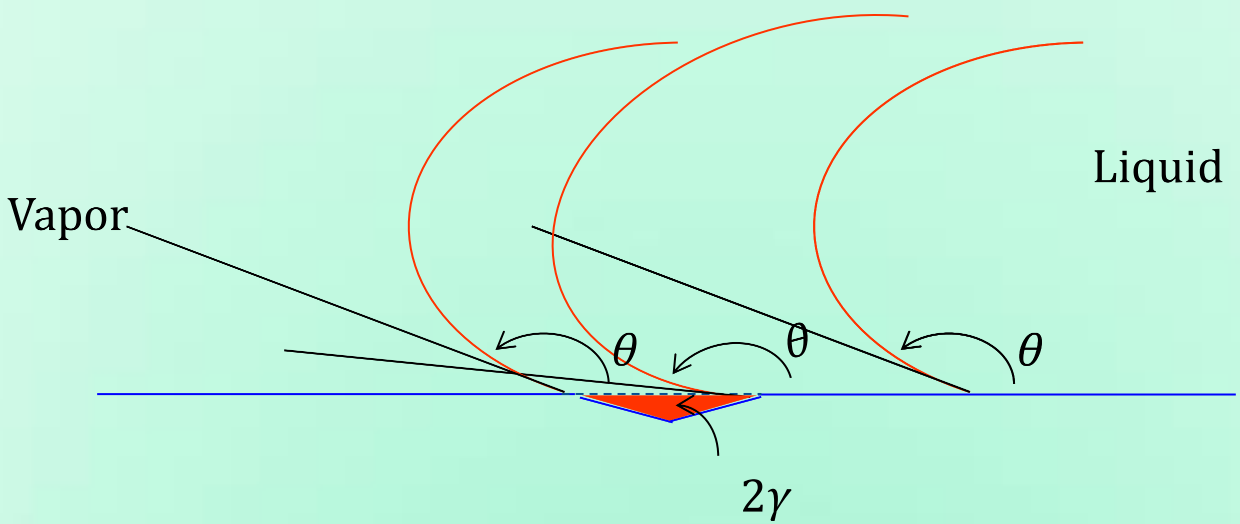
Homogeneous and Heterogeneous Nucleation

- Bubble nucleation completely within a superheated liquid or subcooled vapor is called homogeneous nucleation.
- Nucleation at the interface between a metastable phase and another phase that it contacts is called heterogeneous nucleation.

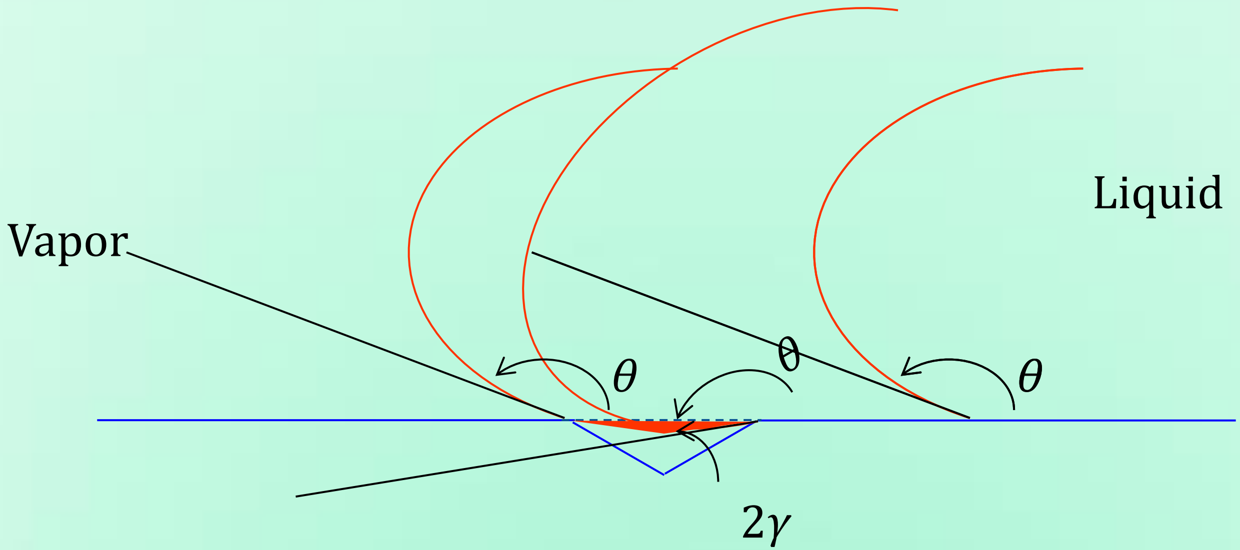
Trapped Vapor or Liquid Pocket



No Vapor Pocket



Trapped Vapor Pocket



Condition for gas entrapment by the advancing front: $\theta_a > 2\gamma$

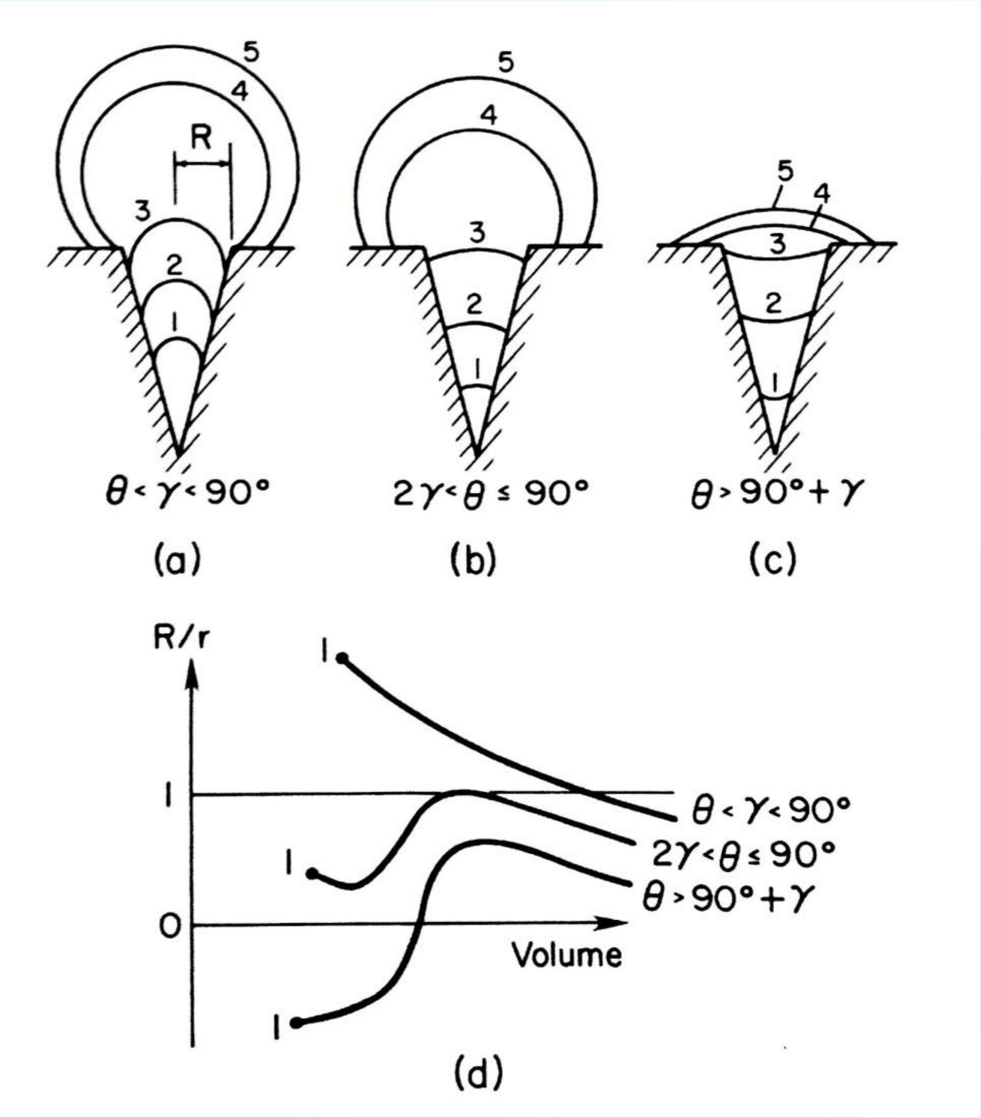
Condition for liquid entrapment in the groove: $\theta_r < 180° - 2\gamma$

Vapor Embryo within and out of Cavity

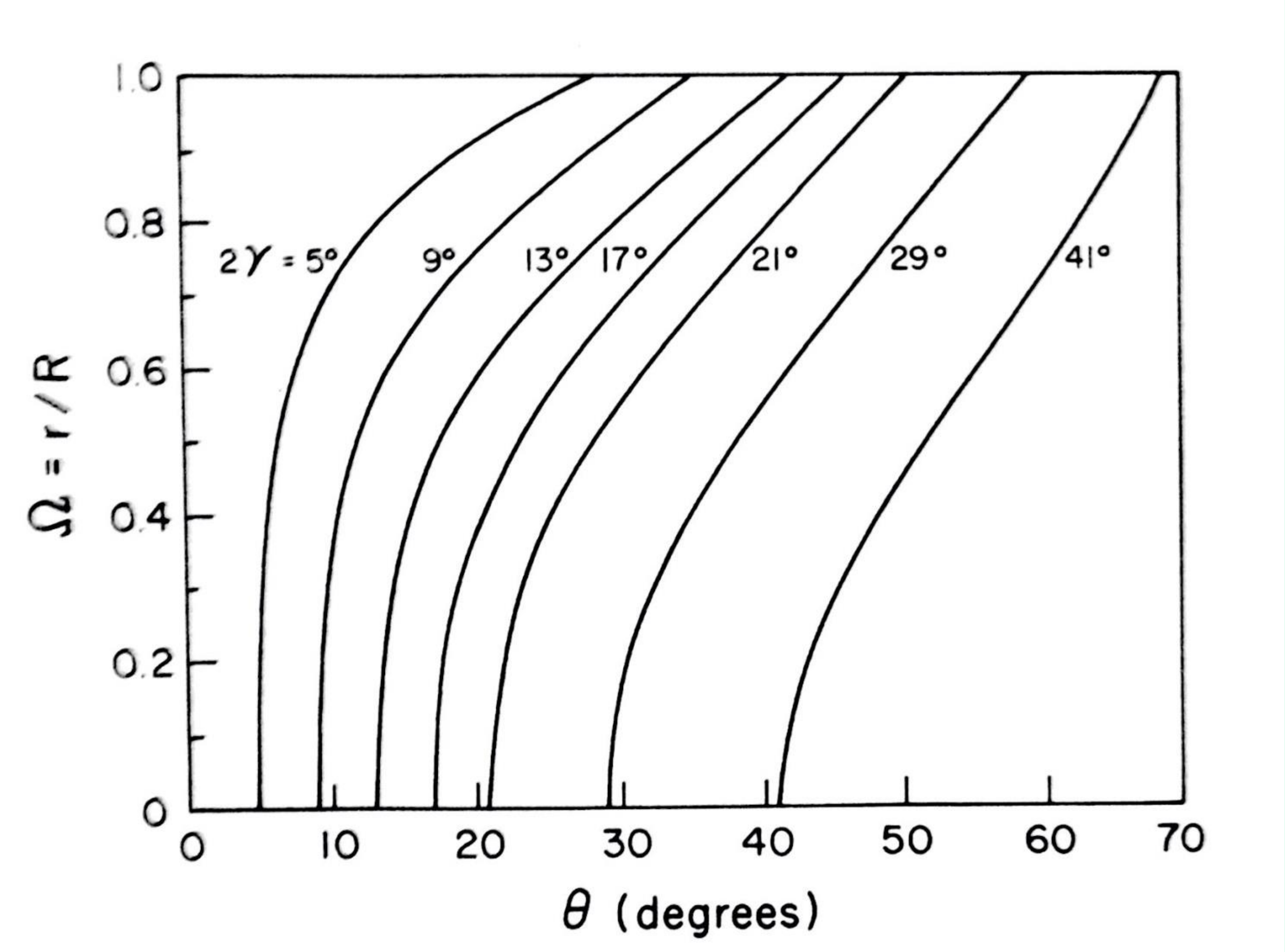
R is mouth cavity

r is embyo R_c

2γ is cone angle (cavity)



Initial *r* of Vapor Embryo with Cavity Cone Angle (γ), θ

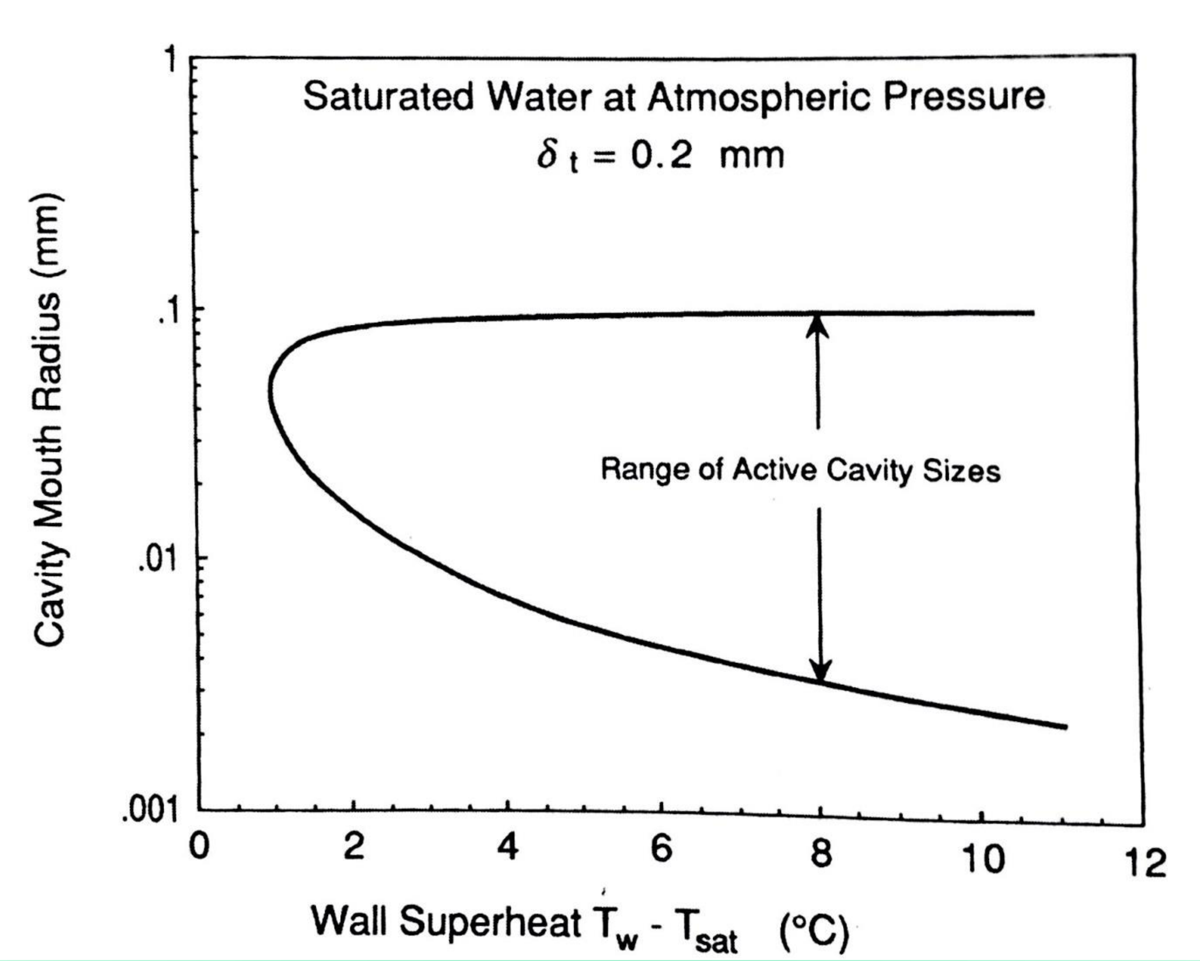


Heterogeneous Nucleation

1. A certain minimum value of wall superheat must be attained before any cavities on the surface will become active nucleation sites
2. Above the superheat required to initiate nucleation, a finite range of cavities can become active sites. The extent of this range depends on the fluid properties, thermal boundary layer, thickness (δ_t) and the subcooling of the bulk fluid.

$$T_l - T_{sat}(P_l) > \frac{2\sigma T_{sat}(P_l) v_{lv}}{h_{lv} R} \text{ for } \frac{R}{r} \leq 1$$

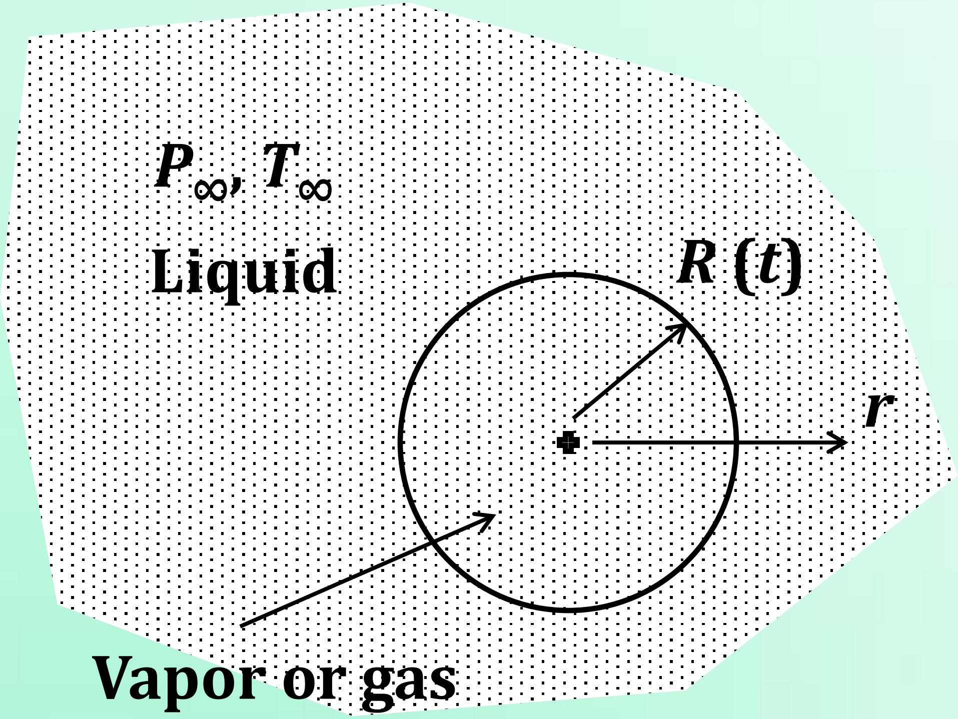
Range of Cavity Sizes



Bubble Growth in a Superheated Liquid

Assumption

A single bubble of spherical shape



Growing Bubble in the bulk of a Liquid

Some characteristics of Bubble Growth

- Initially, the interface temperature close to superheated liquid temperature
- Vapor generated at interface at pressure nearly equal to $P_{sat}\,(T_\infty)$.
- As the temperature of the superheated liquid near the interface reduces with time (Thermal energy is consumed to generate vapor) the liquid temperature reduces towards $T_{sat}\,(P_\infty)$.

- Pressure inside the bubble is high initially, drops gradually.
- As the radius increases & capillary pressure difference decreases.

Some characteristics of Bubble Growth

Temperature & Pressure ranges during growth period are:

$$P_{sat}\left(T_{\infty}\right) \geq P_{v} \geq P_{\infty}$$

$$T_{\infty} \geq T_{v} \geq T_{sat}\left(P_{\infty}\right)$$

Rate of Bubble Growth Dictated by

1. Fluid momentum & Pressure difference
2. Rate of Vaporization which depends on rate of heat transfer
3. Local thermodynamic equilibrium which is assumed to exist at the interface

$$P_v = P_{sat}\left(T_v\right)$$

Two Limiting Cases of Bubble growth

1. Inertia Controlled Bubble Growth

- Initial stage of growth
- Pressure has the Maximum value
- $P_{sat}\,(T_\infty)$ & $T_v \cong T_\infty$
- Heat transfer to the interface is very fast
- Growth rate dictated by momentum transfer, not by rate of heat transfer
- Limited by how rapidly it can push back the surrounding liquid
- Faster rate of growth
- **Initial stage**

Two Limiting Cases of Bubble growth

2. Heat transfer controlled Bubble growth

- Later stage of Bubble growth
- T_v approaches Minimum value
- $T_v \rightarrow T_{sat}\,(P_\infty)$ and $P_v \cong P_\infty$
- Growth is limited by the relatively slower transport of heat to the interface
- Growth is dictated by heat transfer (Energy eq.)
- Momentum transfer between the bubble and surrounding liquid is not a limiting factor
- Slow process
- **Later stage**

Bubble Radius in Inertia Controlled Growth

Initial stage (Inertia controlled):

$$R(t)=\left\{\frac{2}{3}\frac{\left[T_{\infty}-T_{sat}\left(P_{\infty}\right)\right]}{T_{sat}\left(P_{\infty}\right)}\frac{h_{lv}\rho_{v}}{\rho_{l}}\right\}^{\frac{1}{2}}t$$

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Later stage (heat transfer controlled):

$$R(t)=2\sqrt{\frac{3}{\pi}\alpha_l t}\ \ \mathrm{Ja}$$

$$\text{Jakob number, Ja}=\frac{\rho_l c_{pl}\left(T_{\infty}-T_{sat}\right)}{\rho_v h_{lv}}$$

Inertia-Controlled Bubble Growth

Assuming inertia-controlled growth, estimate the interface velocity of a 0.2 mm diameter bubble growing in water at atmospheric pressure and 120°C.

T_{sat} = 100°C, h_{lv} = 2257 kJ/kg, ρ_l = 958 kg/m^3, ρ_v = 0.598 kg/m^3

$$R(t) = \left\{ \frac{2}{3} \frac{\left[T_\infty - T_{sat}(P_\infty)\right]}{T_{sat}(P_\infty)} \frac{h_{lv}\rho_v}{\rho_l} \right\}^{\frac{1}{2}} t$$

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V = 7.1 m/s

Phase Change: Bubble Terminology

1. Ebullition cycle: The steady cyclic growth and release of vapor bubbles at any active nucleation site
2. Bubble departure diameter (d_d) at departure
3. Waiting period (t_w): The time from the bubble depart and the inception of another bubble
4. Frequency at which bubbles are generated and released

Time period (τ) associated with the growth of each bubble, must equal the sum of the waiting period and the time required for the bubble grow to its departure diameter.

$$\frac{1}{f} = \tau = t_w + t_{2R(t)=d_d}$$

Bubble Departure Diameter Correlations

Departure diameter in terms of Bond number

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Zuber suggested

$$f\,d_d = 0.59\left[\frac{\sigma\,g\left(\rho_l - \rho_v\right)}{\rho_l^2}\right]^{0.25}$$

Bubble Departure Frequency

Estimate the bubbling frequency for saturated water at atmospheric pressure for a wall superheat of 20°C.

T_{sat} = 100°C, h_{lv} = 2257 kJ/kg, ρ_l = 958 kg/m^3, ρ_v = 0.598 kg/m^3,

σ = 0.0588 N/m

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Ja = 60
d_d = 6 mm
f = 15.4 Hz
τ = 0.065 s

Estimate the interface velocity of a 0.2 mm diameter bubble growing in water at atmospheric pressure and 120°C.

Estimate the departing diameter and bubble frequency.

T_{sat} = 100°C, h_{lv} = 2257 kJ/kg, ρ_l = 958 kg/m^3, ρ_v = 0.598 kg/m^3

$$\boldsymbol{R(t) = \left\{ \frac{2}{3} \frac{\left[T_\infty - T_{sat}(P_\infty)\right]}{T_{sat}(P_\infty)} \frac{h_{lv}\rho_v}{\rho_l} \right\}^{\frac{1}{2}} t}$$

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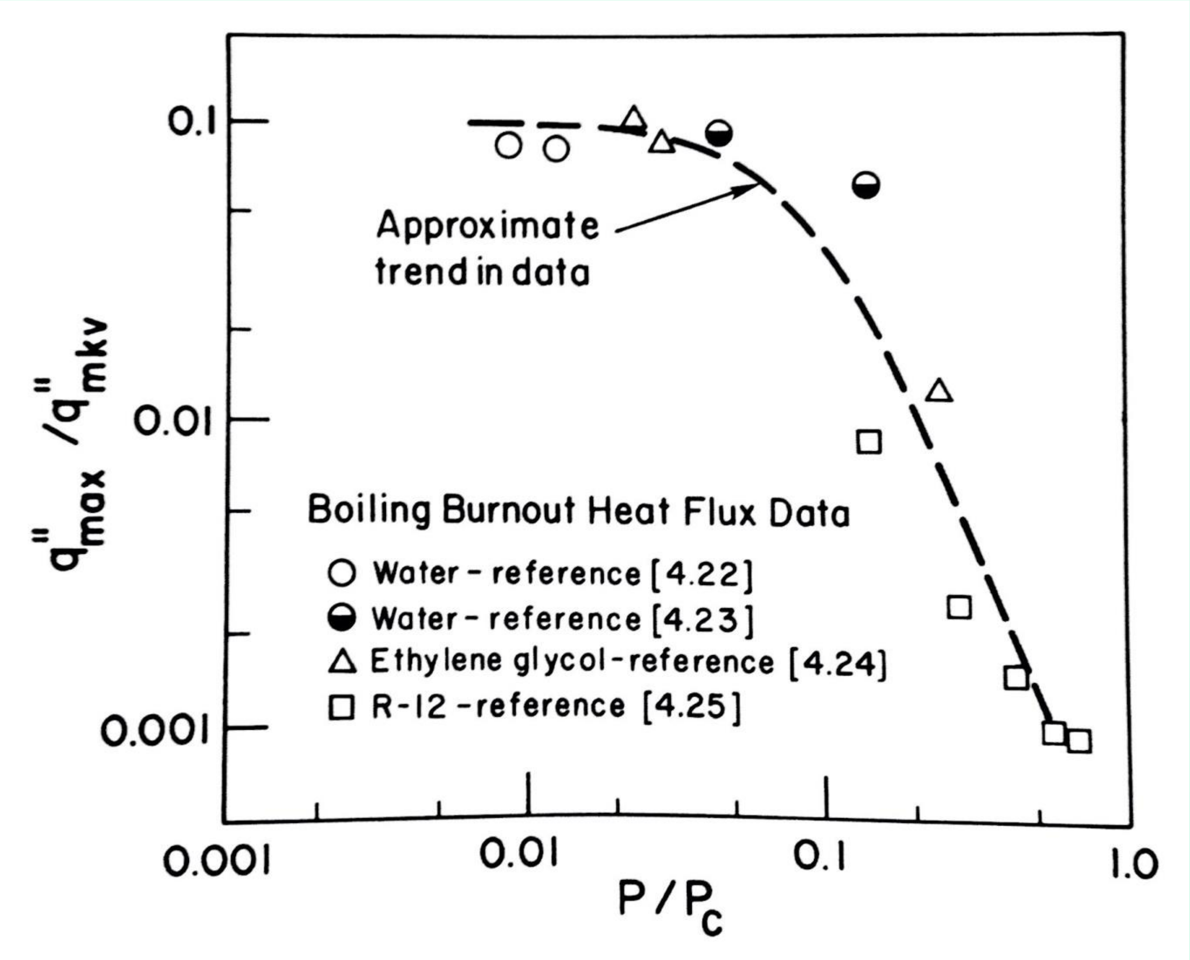
V = 7.1 m/s

Kinetic Theory - Maximum Flux Limitations

- There is an upper limit to the heat flux that can be attained in a phase-change process
- For a vaporization process, the maximum heat flux conceivable would result if molecules were emitted from the interface and no molecules were allowed to return to the liquid.
- Assuming Maxwellian ideal gas,

$$q''_{mkv} = \rho_v h_{lv} \left(\frac{\overline{R} T_v}{2\pi \overline{M}} \right)^{\frac{1}{2}}$$

Maximum Flux vs Burnout



Measured burnout data obtained in different boiling experiments normalized with the corresponding maximum heat flux suggested by kinetic theory

Reasons for Deviation

1. Many molecules leaving the liquid at the interface will return to the liquid by molecular collisions
2. Wall temperatures required to deliver even 10% of q''_{mkv} would result in wall temperatures that exceed the melting point of the material
3. The tendency for more of the heat to find single-phase convection and conduction paths at higher pressures

- Same tendency is applied for condensation but with a different constant

$$q''_{mkv} = 0.741 \rho_v h_{lv} \left(\frac{\overline{R} T_v}{2\pi \overline{M}} \right)^{\frac{1}{2}}$$